DOI: 10.61458/jepm2301018B
UDC: 536.24:621.3.045.15

Journal of
Engineering & Processing Management

ORIGINAL SCIENTIFIC PAPER

Transient thermal behaviour of a substrate subjected to the activation of an electronic chip and surface cooling

Jean-Gabriel Bauzin[1] | Mehdi Belkacem Cherikh[1] | Ali Hocine[1] | Najib Laraqi[1] | Minh-Nhat Nguyen[1]

[1]Université Paris Nanterre, Laboratoire Thermique Interfaces Environnement (LTIE), EA 4415, 50 rue de Sèvres, 92410 Ville d'Avray – France

[2]Faculty of Agriculture Science, Food Industry and Environmental Protection "Lucian Blaga ", university of Sibiu 550012 Sibiu Romania

Correspondence
Jean-Gabriel Bauzin, Université Paris Nanterre, Laboratoire Thermique Interfaces Environnement (LTIE), EA 4415, 50 rue de Sèvres, 92410 Ville d'Avray – France
Email: jbauzin@parisnanterre.fr

Abstract

Controlling the temperature of electronic components is a major interest for the electronics industry. Indeed, the lifetime of the components is directly dependent on the temperature levels reached in the electronic boards. Then, it is essential to predict the chip temperature evolution in order to maximize their lifespan. The electronic boards are more and more complex. They are multi-layers composed of different materials. The numerical resolution of the heat transfer equations in these systems requires very fine meshes and therefore very high computation times. It is possible to standardize the characteristics of these multilayer boards in order to treat them as a homogeneous material. The study presented in this work uses this approach and deals with the transient thermal behaviour of a substrate and its chip. The entire surface of the electronic board is cooled by convection. The developed model assumes that the surface convection coefficient is known, constant and uniform. The heat transfer by conduction in the substrate is based on an axisymmetric assumption on the longitudinal dimensions of the exchange surface (r, theta) and an assumption of semi-infinite medium in the transversal direction of the plate (thickness z). These assumptions are verified if, on the one hand, the activation times of the electronic chips are low enough and the dimensions of the chip is small compared to the electronic board. In these conditions, a fully analytical model is developed considering two successive integral transforms: a Laplace transform for the temporal variable, and a Hankel transform for the radial variable. An explicit expression of the temperature of the surface heated by the component is established, requiring very short computation times compared to numerical simulations. This model can be easily incorporated into a dimensioning code for electronic devices to predict their temperature. It can also be used as a direct model in an inverse procedure for identifying parameters on electronic boards.

Keywords: analytical thermal computation, cooling of electronic systems, integral transforms

1. INTRODUCTION

The cooling of electronic components has a major industrial aspect. Indeed, the knowledge and control of the temperature of electronic chips is essential in order to control the life of the latter. Many studies address the thermal behaviour in the substrate whether in a variable or steady state by analytical approaches(Monier-Vinard, Laraqi, Dia, Nguyen, & Bissuel 2013; 2016; Monier-Vinard, Nguyen, Laraqi, Bissuel, & Daniel 2016; Nguyen, Monier-Vinard, Laraqi, & Bissuel 2016). The numerical solutions require particularly high computation times due to the numerical mesh required for their implementation.

In this study, we propose to analyse the thermal behaviour of a substrate subjected to the heating of a circular chip, the whole being cooled by convection. The substrate will be considered isotropic. This configuration is similar to other thermal systems studied in the literature such as media subjected to a laser flux or a heat source (Chen 2017; Chen & Bi 2017; Laraqi 2010; Laraqi, Alilat, de Maria, & Baïri 2009). It can also represent the cooling of a radiator of a LED (Janicki, Torzewicz, Samson, Raszkowski, & Napieralski 2018). For the reasons mentioned above, the study carried out here is analytical and establishes the explicit expression of the temperature evolution in the case of a simple power dissipation scenario by the chip. Finally, a convolution calculation is used to establish the temperature response of the system whatever the scenario for the power dissipated by the chip.

2. MATERIALS AND METHODS

The system studied is an assembly consisting of a substrate with a circular chip positioned on its surface. This chip generates a heat power that will heat the substrate, the whole being cooled by convection on the surface of the assembly. Assuming that the chip has a very small thickness and that the contact resistance between the chip and the substrate is very low, the thermal system can be represented as shown in Figure 1.

Since the system is cylindrical, the heat equation is written:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

We assume an isotropic conductivity in the substrate, but the case of anisotropic conductivity does not pose any particular difficulties. At the initial time, the whole system is at a uniform temperature, equal to the ambient temperature. In the rest of the problem, we will calculate temperature increases with respect to the ambient temperature (this is equivalent to assuming a zero temperature environment as noted in Figure 1. The initial condition of the problem is then written:

$$\boldsymbol{T}(\boldsymbol{r}, \boldsymbol{z}, 0) = 0 \tag{2}$$

The substrate is subjected to heat flux over the chip area and convection over the entire surface. The boundary conditions are written as follows:

$$-\lambda\left(\frac{\partial T}{\partial z}\right)_{r,z=0,t} = \begin{cases} \varphi(t) & 0 < r < a \\ -hT(t) & 0 < b \end{cases} \tag{3}$$

In the direction of the substrate depth, the medium is considered to be semi-infinite. This assumption is respected because for the calculation times studied, the heat front does not reach the lower limit of the substrate. Taking into account a finite thickness does not pose any particular difficulty. Then, the boundary condition of the semi-infinite medium is written as:

$$\boldsymbol{T}(\boldsymbol{r}, \boldsymbol{z} \to +\infty, \boldsymbol{t}) = 0 \tag{4}$$

The axisymmetry and lateral adiabaticity conditions of the system are written respectively:

$$\left(\frac{\partial T}{\partial r}\right)_{r=0,z} = 0, \quad \left(\frac{\partial T}{\partial r}\right)_{r=b,z} = 0 \tag{5}$$

In order to solve the problem, we will perform two successive integral transformations, one on time, the other on space. First, the Laplace Transform is used on the time variable:

$$\overline{T} = \int_0^{+\infty} T e^{-pt} dt \tag{6}$$

In the second step, we use the finite Hankel integral transform along the direction of the ray such that:

$$\tilde{T} = \int_0^b r T J_0(\beta r) dr \tag{7}$$

The physical equations of the problem (1 to 5) then become, after transformations:

$$\frac{\partial^2 \tilde{\overline{T}}}{\partial z^2} - \left(\beta_n^2 + \frac{p}{\alpha}\right)\tilde{\overline{T}} = 0 \tag{8}$$

Withβ_n being solutions of the following transcendental equation:

$$J_1(\beta_n b) = 0 \tag{9}$$

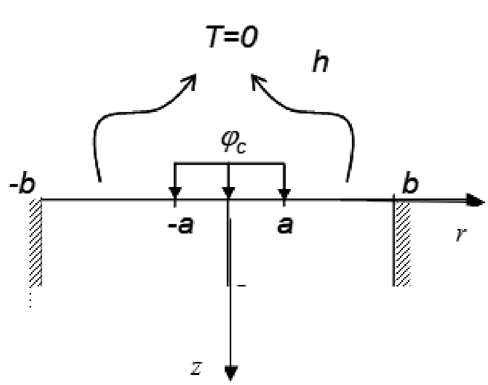

Figure 1. Diagram of the thermal problem.

The boundary conditions become:

$$-\lambda\left(\frac{\partial\tilde{\tilde{\overline{T}}}}{\partial z}\right)_{z=0}=\tilde{\tilde{\overline{\varphi}}}-h\left(\tilde{\tilde{\overline{T}}}\right)_{z=0}\tilde{\tilde{\overline{T}}}_{z\to+\infty}=0 \quad (10)$$

With

$$\tilde{\tilde{\overline{\varphi}}}=\frac{\overline{\varphi}a}{\beta_n}J_1(\beta_n a) \quad (11)$$

The general form of the temperature solution is then:

$$\tilde{\tilde{\overline{T}}}_n=A e^{z\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}}+B e^{-z\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}} \quad (12)$$

Using the boundary condition$\tilde{\tilde{\overline{T}}}_{z\to+\infty}=0$, we deduce that$A=0$. Thus, the expression for the transformed temperature is:

$$\tilde{\tilde{\overline{T}}}_n=B e^{-z\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}} \quad (13)$$

The$\beta_n$values can be calculated using the McMahon relation, which does not require searching for the zeros of the transcendental equation (Equation (2)). Then, we have:

$$\tilde{\tilde{\overline{T}}}=\tilde{\tilde{\overline{T}}}_0+\sum_{n=1}^{+\infty}\tilde{\tilde{\overline{T}}}_n \quad (14)$$

- Case *n=0*

$$\tilde{\tilde{\overline{T}}}_0=\frac{\overline{\varphi}a^2}{2\left(\lambda\sqrt{\frac{p}{\alpha}}+h\right)} \quad (15)$$

- Case *n≠0*

Applying the boundary condition at*z=0* of (Equation (10)), we obtain the following relation:

$$B\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}=\frac{\overline{\varphi}aJ_1(\beta_n a)}{\beta_n}-hB \quad (16)$$

Therefore, for the expression of the constant$B$:

$$B=\frac{\overline{\varphi}aJ_1(\beta_n a)}{\beta_n\left[h+\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]} \quad (17)$$

The expression for the temperature transform is then given by the following relation:

$$\tilde{\tilde{\overline{T}}}_n=\frac{\overline{\varphi}aJ_1(\beta_n a)}{\beta_n\left[h+\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]}e^{-z\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}} \quad (18)$$

Applying the inverse Hankel transform:

$$\overline{T}=H^{-1}\left(\tilde{\tilde{\overline{T}}}\right)=\frac{2}{b^2}\sum_{n=0}^{\infty}\frac{J_0(\beta_n r)}{J_0^2(\beta_n b)}\tilde{\tilde{\overline{T}}} \quad (19)$$

The expression for the temperature in the Laplace domain is:

$$\overline{T}(r,z,p)=\frac{\overline{\varphi}a^2}{b^2\left(\lambda\sqrt{\frac{p}{\alpha}}+h\right)}+\frac{2\overline{\varphi}a}{b^2}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)e^{-z\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}}}{\beta_n J_0^2(\beta_n b)\left[h+\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]} \quad (20)$$

The surface temperature (*z=0*) in the Laplace domain is expressed as:

$$\overline{T}(r,0,p)=\overline{\varphi}\left(\frac{a^2}{b^2\left(\lambda\sqrt{\frac{p}{\alpha}}+h\right)}+\frac{2a}{b^2}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)\left[h+\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]}\right. \quad (21)$$

The expression of the temperature in the time domain will depend on the temporal evolution of the dissipated power in the chip. First, we focus on a step power applied to the chip. Then, we study the temperature response to a flux pulse. This will allow us to calculate, by convolution, the temperature response to any power scenario.

For a flux step, we have:

$$\overline{\varphi}=\frac{\varphi_c}{p} \quad (22)$$

The expression for the temperature$T_e$in the Laplace domain for a step input is:

$$\overline{T}_e(r,0,p)=\frac{\varphi_c a^2}{b^2p\left(\lambda\sqrt{\frac{p}{\alpha}}+h\right)}+\frac{2a\varphi_c}{b^2}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)p\left[h+\lambda\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]} \quad (23)$$

Applying the inverse Laplace transform, we obtain the explicit temperature at a given radius and any given time$t$:

$$T_e(r,0,t)=\frac{\varphi_c a}{b^2}\left(\frac{a}{h}\left(1-erfc\left(\frac{h}{\lambda}\sqrt{\alpha t}\right)e^{\left(\frac{h}{\lambda}\right)^2\alpha t}\right)+2\sum_{n=1}^{\infty}\left[\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)\lambda\left(\beta_n^2-\left(\frac{h}{\lambda}\right)^2\right)}\times\left(\frac{h}{\lambda}\left(erfc\left(\frac{h}{\lambda}\sqrt{\alpha t}\right)e^{\left(\left(\frac{h}{\lambda}\right)^2-\beta_n^2\right)\alpha t}-1\right)+\beta_n erf\left(\beta_n\sqrt{\alpha t}\right)\right)\right.\right. \quad (24)$$

The impulse response of the system refers to the temperature response of the system to an impulse input. The input flux in Laplace space for an impulse response is simply given by:

$$\overline{\varphi}=1 \quad (25)$$

The expression of the impulse response in temperature in Laplace space is given by:

$$\overline{T}_i(r,0,p)=\left(\frac{a^2}{\lambda b^2\left(\sqrt{\frac{p}{\alpha}}+\frac{h}{\lambda}\right)}+\frac{2a}{\lambda b^2}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)\left[\frac{h}{\lambda}+\sqrt{\left(\beta_n^2+\frac{p}{\alpha}\right)}\right]}\right. \quad (26)$$

The expression of the impulse response in space-time is obtained after applying the inverse Laplace transform.

$$T_i(r,0,t) = \left(\frac{\alpha a^2}{\lambda b^2}\left(\frac{1}{\sqrt{\alpha\pi t}} - \frac{h}{\lambda}e^{\alpha\left(\frac{h}{\lambda}\right)^2 t} erfc\left[\frac{h}{\lambda}\sqrt{\alpha t}\right]\right)\right) \times \left[1 + \frac{2}{a}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)}e^{-\alpha\beta_n^2 t}\right] \quad (27)$$

The temperature response to any input signal can be calculated by convolving the impulse response (Equation (27)) with the power dissipation evolution of the electronic chip $\varphi(t)$. This is given by the following expression (Al Hadad, Maillet, & Jannot 2018; Bauzin, Nguyen, Laraqi, Vaca Hernández, & Dehmani 2019; Howell 2016; Peyrière 2012):

$$T(r,0,t) = \int_{-\infty}^{+\infty} \left(\frac{\alpha a^2}{\lambda b^2}\left(\frac{1}{\sqrt{\alpha\pi t}} - \frac{h}{\lambda}e^{\alpha\left(\frac{h}{\lambda}\right)^2 t} erfc\left[\frac{h}{\lambda}\sqrt{\alpha t}\right]\right)\right) \times \left[1 + \frac{2}{a}\sum_{n=1}^{\infty}\frac{J_1(\beta_n a)J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)}e^{-\alpha\beta_n^2 t}\right.\varphi(t-\xi)\,d\xi \quad (28)$$

3. RESULTS AND DISCUSSION

The analytical expressions given by Equations 24,(27) and (28) allow us to plot temperature profiles or evolutions on the surface of the substrate. For the rest of the study, the power dissipated by the electronic chip will be taken as 1 W. Likewise, the convection coefficient will be taken as constant and equal to 10 W/m^2K. Table 1 presents the thermal characteristics of the material used for the substrate, as well as its dimensions. These values can be adapted depending on the substrates studied.

Table 1. Thermal and geometrical characteristics of the substrate

$\gamma[W/mK]$	$\alpha 10^{-7}[m/s]$	a [mm]	b [mm]
1	2	2	100

Figure 2 shows the evolution of the temperature at the center of the chip (r=0) when the dissipated power is constant (Equation (24)). This is the maximum temperature (dimensioning) in the system.

Using the same expression, it is possible to plot the temperature profile as a function of radius at the surface of the substrate (Figure 3) for a given time (here$t$=$20s$). It can be observed that outside the electronic chip area (r=$2mm$), the maximum temperature increase at the surface represents only about one-third of the maximum increase calculated at the center of the system.

The temperature impulse response of the substrate is necessary to calculate the temperature evolution for any power dissipation scenario in the electronic chip. Figure 4 represents this response (Equation (27)).

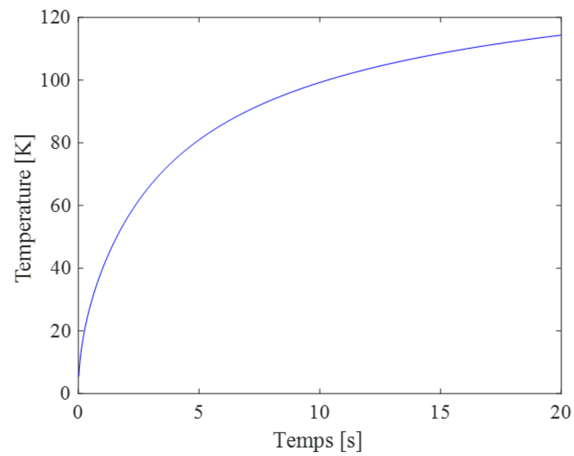

Figure 2. Evolution of the maximum temperature of the component.

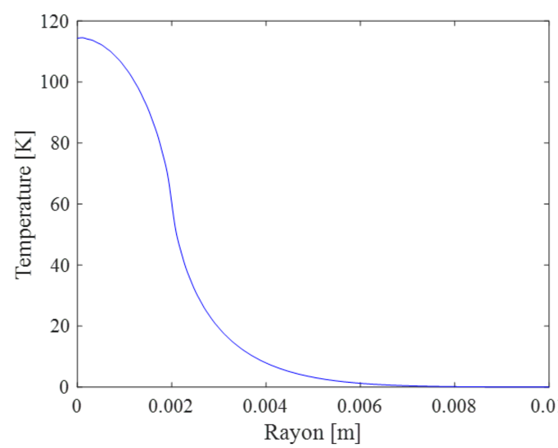

Figure 3. Temperature distribution across the radius for t=20s.

Thus, it is possible to calculate, for example, the response to a step-like power configuration (Figure 5).

It is interesting to note that for an equivalent dissipated energy over a given time, the shape of the power evolution over time strongly impacts the maximum temperature reached in the component (Figure 6). Thus, power peaks are very detrimental to the chip's durability, resulting in high temperature levels. For "critical" systems, it will be important to regulate power calls to the electronic device in order to avoid damaging them.

In this figure, the temperature evolution of the substrate for a pulsed power dissipation (blue curve) and for an equivalent constant average power dissipation (red curve) is compared. The constant power is chosen such that it dissipates the same amount of energy as the pulsed power over the same time period. It can be seen that the temperature evolution for the pulsed power is more dy-

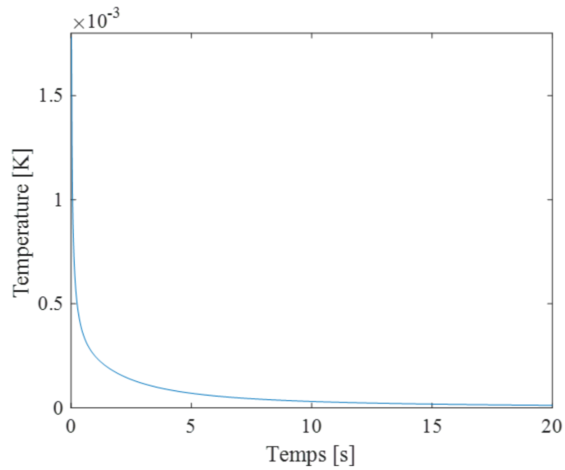

Figure 4. The impulse response in temperature of the substrate at r=0.

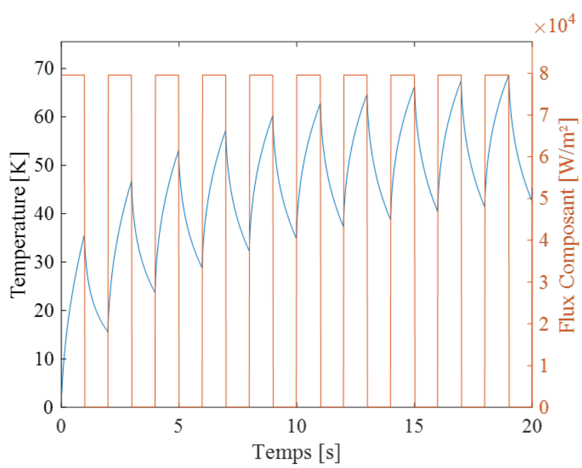

Figure 5. Evolution of the temperature of the substrate for a square-wave power supply at r=0.

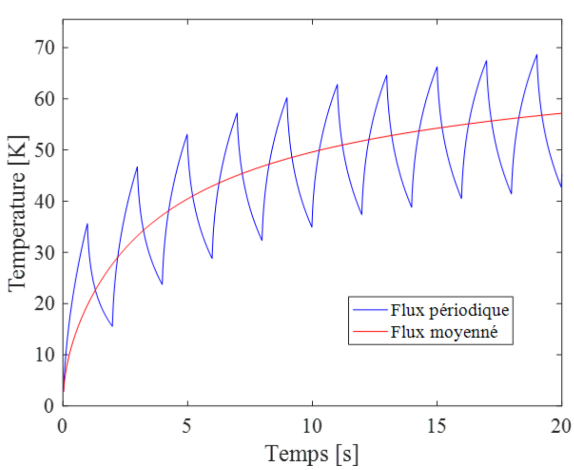

Figure 6. Comparison of the temperature evolution of the substrate for a pulsed power dissipation and for an equivalent constant average power dissipation at r=0.

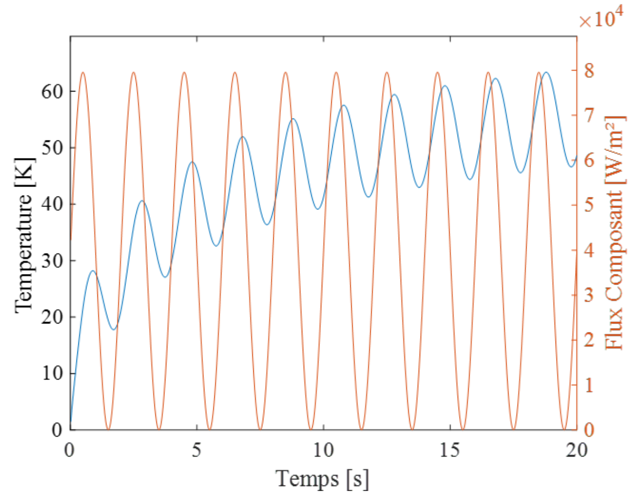

Figure 7. The evolution of the substrate temperature for a sinusoidal power input at the center of the component (r=0).

namic and has a higher peak temperature than the equivalent constant power. This demonstrates the importance of considering the temporal behaviour of the power dissipation when analysing the thermal behaviour of electronic components.

Similarly, it is possible to calculate the temporal response to a sinusoidal dissipated power (Figure 7). The calculation can be applied to any scenario of power dissipation in the electronic chip.

4. CONCLUSIONS

This study has established analytical relationships for calculating the temporal response of a substrate subjected to the activation of an electronic chip, while being cooled by convection. This calculation uses Laplace and Hankel integral transforms. The explicit expression of the surface temperature has been developed in the case of a constant dissipated power. The study also made it possible to calculate the temperature response of the system for any power dissipation scenario in the component using the convolution product. These results can be integrated into thermal calculation codes for electronic devices. In view of an experimental validation of these analytical results, an infrared measurement of surface temperatures for different power activation scenarios of the chip is being considered. Finally, a development regarding the consideration of a non-isotropic substrate could be the subject of a future study.

5. ACKNOWLEDGMENTS

This work was supported by Doctoral School of "Connaissance, Langage, Modélisation" of University Paris Nanterre.

REFERENCES

Al Hadad, W., Maillet, D., & Jannot, Y. (2018). Experimental transfer functions identification: Thermal impedance and transmittance in a channel heated by an upstream unsteady volumetric heat source. *International Journal of Heat and Mass Transfer*, *116*, 931–939. http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.09.079

Bauzin, J.-G., Nguyen, M.-N., Laraqi, N., Vaca Hernández, A., & Dehmani, A. (2019). Thermoelastic mechanical and heat conduction study through inverse method and transfer functions. *International Journal of Heat and Mass Transfer*, *135*, 1260–1268. http://dx.doi.org/10.1016/j.ijheatmasstransfer.2019.02.049

Chen, G. (2017). Axisymmetric modeling of long pulsed laser heating with convective boundary conditions using analytical solutions. *Optik*, *130*, 1038–1044. http://dx.doi.org/10.1016/j.ijleo.2016.11.110

Chen, G., & Bi, J. (2017). A semi-analytical solution for 2-d axisymmetric modeling of repetitive pulse laser heating with body absorption. *International Journal of Heat and Mass Transfer*, *111*, 367–373. http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.04.014

Howell, K. B. (2016). *Principles of fourier analysis*. CRC Press.

Janicki, M., Torzewicz, T., Samson, A., Raszkowski, T., & Napieralski, A. (2018). Experimental identification of led compact thermal model element values. *Microelectronics Reliability*, *86*, 20–26. http://dx.doi.org/10.1016/j.microrel.2018.05.003

Laraqi, N. (2010). Thermal impedance and transient temperature due to a spot of heat on a half-space. *International Journal of Thermal Sciences*, *49*(3), 529–533. http://dx.doi.org/10.1016/j.ijthermalsci.2009.10.004

Laraqi, N., Alilat, N., de Maria, J. G., & Baïri, A. (2009). Temperature and division of heat in a pin-on-disc frictional device—exact analytical solution. *Wear*, *266*(7–8), 765–770. http://dx.doi.org/10.1016/j.wear.2008.08.016

Monier-Vinard, E., Laraqi, N., Dia, C., Nguyen, M., & Bissuel, V. (2013). Analytical thermal modelling of multilayered active embedded chips into high density electronic board. *Thermal Science*, *17*(3), 695–706. http://dx.doi.org/10.2298/TSCI120826072M

Monier-Vinard, E., Laraqi, N., Dia, C., Nguyen, M.-N., & Bissuel, V. (2016). Analytical modeling of multi-layered printed circuit board using multi-stacked via clusters as component heat spreader. *Thermal Science*, *20*(5), 1633–1647. http://dx.doi.org/10.2298/TSCI140403143M

Monier-Vinard, E., Nguyen, M.-N., Laraqi, N., Bissuel, V., & Daniel, O. (2016, May). Steady-state temperature solution for early design of annealed pyrolytic graphite heat spreader: Full results. In *2016 15th ieee intersociety conference on thermal and thermomechanical phenomena in electronic systems (itherm)*. IEEE. http://dx.doi.org/10.1109/ITHERM.2016.7517647

Nguyen, M.-N., Monier-Vinard, E., Laraqi, N., & Bissuel, V. (2016, November). Effect of heat source orientation on the thermal behavior of n-layer electronic board. *International Journal of Thermal Sciences*, *109*, 23–32. http://dx.doi.org/10.1016/j.ijthermalsci.2016.05.026

Peyrière, J. (2012). *Convolution, series and fourier integrals*. Ellipses.